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AN ASYMPTOTICALLY ORTHONORMAL
POLYNOMIAL FAMILY

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ABSTRACT

Given a Jordan curve Γ in the complex plane, we describe a polynomial family which is asymptotically orthonormal on Γ. The polynomials have some similarities with the Faber polynomials but are simpler to compute with. Numerical examples are presented.

SIGNIFICANCE AND EXPLANATION

Let Γ be a Jordan curve in the complex plane. We describe a polynomial family, which is asymptotically orthonormal on Γ (as the degree of the polynomials increases), and which is simple to compute. We use the polynomials to approximate functions $f(z)$ analytic in the interior of Γ and continuous on Γ. After some initial calculations, which are independent of the functions to be approximated, each functions $f(z)$ can be approximated by an nth degree polynomial in $O(n \log(n))$ operations. Theoretically, one could use the Faber polynomials for Γ, but the proposed polynomials are much simpler to compute with and almost as effective, as is seen by the bound for the resulting polynomial projection given.

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1. INTRODUCTION

Let Γ be a Jordan curve in the complex plane, and let $z = \phi(w)$ be an analytic function on $|w| > 1$, such that ϕ maps $|w| > 1$ onto the exterior of Γ, with $\phi(\infty) = \infty$. ϕ can be extended to a continuous bijective map on $|w| \geq 1$. The polynomials we will study are defined by

$$(1.1) \qquad p_n(z) := \frac{1}{2c^n} \left(\prod_{k=0}^{n-1} (z - \phi(e^{2ki\pi/n})) + \prod_{k=0}^{n-1} (z - \phi(e^{(2k+1)i\pi/n})) \right), \quad n = 0,1,2,\ldots,$$

where c denotes the capacity of Γ.

Ex. 1.1. Let Γ be the unit circle, and let $z = \phi(w) = w$. Then

$$p_n(z) = \frac{1}{2} \left(\prod_{k=0}^{n-1} (z - e^{2ki\pi/n}) + \prod_{k=0}^{n-1} (z - e^{(2k+1)i\pi/n}) \right) =$$

$$= \frac{1}{2} ((z^n + 1) + (z^n - 1)) = z^n, \quad n = 0,1,2,\ldots,$$

These are the Faber polynomials for the unit disk. ∎

Ex. 1.2. Let Γ be the ellipse $\{x + iy, \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1\}$, $a > b$. Let $\alpha := \frac{a+b}{a-b}$ and $d := (a^2 - b^2)^{1/2}$. Then

$$(1.2) \qquad z = \phi(w) = d\alpha(w + \alpha^{-2}w^{-1}) .$$

The capacity of Γ is $d\alpha$. Substitute (1.2), $w_\ell := e^{i\pi\ell/n}$ and $c = d\alpha$ into (1.1). This gives after some simplifications

$$p_n(z) = w^n + \alpha^{-2n}w^{-n} .$$

These are the Faber polynomials for the ellipse bounded by Γ, c.f. Curtiss [2]. For future reference, we note that

(1.3)
$$p_n(z) + w^n, \quad n + \infty$$

if $\alpha > 1$, or equivalently, if $b > 0$. ∎

In section 2, we show that for a large class of boundaries Γ the polynomials $p_n(z)$ are asymptotically orthonormal with respect to an inner product, c.f. (1.2) and (1.3). We use these polynomials to define a bounded projection operator for polynomial approximation of functions, which are analytic interior to Γ and satisfy certain smoothness properties on Γ. In section 3, we show how this projection onto polynomials of degree $< n$ can be computed in $O(n \log(n))$ operations for each function to be approximated, provided that some initial calculations independent of the function to approximate have been carried out. Section 4 contains numerical examples.

2. SOME PROPERTIES OF THE POLYNOMIALS

Let Ω denote the open interior of Γ, and let Ω_c be the complement of Ω.

Theorem 2.1.

Assume that $\dfrac{d^j\phi}{dw^j}$ is continuous on $|w| = 1$, and that $\dfrac{d^{j+1}\phi}{dw^{j+1}}$ is of bounded variation on $|w| = 1$ for some $j > 0$. Then

(2.1) $\qquad p_n(z) = w^n(1 + o(n^{-j}))$, $\quad n \to \infty$, uniformly for $z \in \Omega_c$, where
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad w$ is defined by $\phi(w) = z$.

(2.2) $\qquad p_n(z) = o(n^{-j-1})$, $\qquad n \to \infty$, $z \in \Omega$, and uniformly for z
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ belonging to any closed subset of Ω.

If ϕ is analytic on $|w| = 1$, then there is a constant r, $0 < r < 1$, such that

$\qquad\qquad p_n(z) = w^n + O(r^{-n})$, $\qquad n \to \infty$, uniformly for $z \in \Omega_c$, where
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad w$ is defined by $w = \phi(z)$.

$\qquad\qquad p_n(z) = O(r^{-n})$, $\qquad\qquad n \to \infty$, $z \in \Omega$, and uniformly for z
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ belonging to any closed subset of Ω.

Proof. In the proof we make use of results Curtiss [2] obtained in his investigation of the product

$$\prod_{k=0}^{n-1} (z - \phi(e^{2\pi ik/n})) .$$

Curtiss [2], Lemma 1, shows that if $f(\theta)$ is a 2π-periodic complex valued function of the real variable θ, absolutely continuous on the interval $0 < \theta < 2\pi$, and if $\theta_k = 2\pi\dfrac{k}{n}$, $k = 0(1)n$, then

(2.3) $\qquad\qquad \dfrac{2\pi}{n}\sum_{k=0}^{n-1} f(\theta_k) = \int_0^{2\pi} f(\theta)d\theta + o(n^{-1})$, $\quad n \to \infty$.

If $\dfrac{d^j f}{d\theta^j}$ is absolutely continuous, Curtiss's proof of (2.3) supplemented by integration by parts yields

$$(2.4) \qquad \frac{2\pi}{n} \sum_{k=0}^{n-1} f(\theta_k) = \int_0^{2\pi} f(\theta)d\theta + o(n^{-j-1}), \quad n \to \infty .$$

Following Curtiss, we introduce

$$q(\hat{w},w) := \begin{cases} \dfrac{\phi(\hat{w}) - \phi(w)}{c(\hat{w} - w)}, & \hat{w} \neq w \\ \\ \phi'(w) , & \hat{w} = w , \end{cases}$$

where $|w| > 1$, $|\hat{w}| = 1$. Let $\psi(\theta,w) := \log(q(e^{i\theta},w))$. With a branch of the logarithm choosen so that $w \to \psi$ is analytic and single valued for $|w| > 1$, continuous on $|w| = 1$ and vanishes as $|w| \to \infty$, the Cauchy integral formula yields

$$\int_0^{2\pi} \psi(\theta,w)d\theta = 0, \quad |w| > 1 .$$

Let $\theta_k := 2\pi k/n$, $k = 0(1)n - 1$. Then, with $z = \phi(w)$,

$$(2.5) \qquad \frac{\prod_{0}^{n-1} (z - \phi(e^{i\theta_k}))}{c^n(w^n - 1)} = \prod_{0}^{n-1} \left(\frac{z - \phi(e^{i\theta_k})}{c(w - e^{i\theta_k})}\right) = \prod_{0}^{n-1} q(w,e^{i\theta_k}) .$$

With a suitable branch of the logarithms, we have

$$\log\left(\frac{\prod_{k=0}^{n-1} (z - \phi(e^{i\theta_k}))}{c^n(w^n - 1)}\right) = \sum_{k=0}^{n-1} \psi(w,\theta_k) = \frac{n}{2\pi} \sum_{k=0}^{n-1} \psi(w,\theta_k)(\theta_{k+1} - \theta_k) =$$

$$(2.6) \qquad = \frac{n}{2\pi} \left(\sum_{k=0}^{n-1} \psi(w,\theta_k)(\theta_{k+1} - \theta_k) - \int_0^{2\pi} \psi(w,\theta)d\theta\right) =: H_n(z) .$$

Hence

$$(2.7) \qquad \frac{1}{c^n} \prod_{k=0}^{n-1} (z - \phi(e^{i\theta_k})) = (w^n - 1)(1 + O(H_n(z))), \quad n \to \infty$$

Curtiss [1] considers the case $j = 0$, and shows that $H_n(\phi(w)) = o(1)$, $n \to \infty$, uniformly for $|w| > 1$. For $j > 0$, some straightforward modifications of Curtiss's proof, like

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replacing (2.3) by (2.4), yields $H_n(\phi(w)) = o(n^{-j})$, $n \to \infty$ uniformly for $|w| \geq 1$. If

$\phi(w)$ is analytic in a neighborhood of the unit circle, then ψ is an analytic function

of both its arguments, and $|H_n(\phi(w))| < Ar^n$ for some constants A and r, $0 < r < 1$ as

$n \to \infty$, uniformly for $|w| \geq 1$.

Now replace θ_k by $\hat{\theta}_k := \theta_k + \frac{\pi}{n}$, $k = 0(1)n - 1$, in (2.5). Then

$$(2.8) \qquad \prod_{k=0}^{n-1} q(w, e^{i\hat{\theta}_k}) = \frac{\prod_{k=0}^{n-1} \left(z - \phi(e^{i\hat{\theta}_k})\right)}{c^n(w^n + 1)},$$

and analogously to (2.7) we obtain

$$(2.9) \qquad \frac{1}{c^n} \prod_{k=0}^{n} (z - \phi(e^{i\hat{\theta}_k})) = (w^n + 1)(1 + O(H_n(z))), \quad n \to \infty.$$

The average of (2.7) and (2.9) yields (2.1). Also (2.2) follows from results of

Curtiss. For ϕ of bounded variation on $|w| = 1$, Curtiss shows that

$$(2.10) \qquad \frac{1}{c^n} \prod_{k=0}^{n-1} (z - \phi(e^{i\theta_k})) = -1 + O(h(n)), \; h(n) = o(1), \; n \to \infty,$$

for any $z \in \Omega$, and uniformly for z belonging to any closed subset of Ω. Again it is

straightforward to show that if $\dfrac{d\phi_j}{dw^j}$ is of bounded variation for some $j > 0$, then

$h(n) = o(n^{-j-1})$. If ϕ is analytic in a neighborhood of $|w| = 1$, there are constants

$B, r, 0 < r < 1$, such that $|h(n)| < Br^{-n}$. Inspection of Curtiss proof also shows that

$$(2.11) \qquad \frac{1}{c^n} \prod_{k=0}^{n-1} (z - \phi(e^{i\theta_k})) = 1 + O(h(n)), \quad n \to \infty,$$

for any $z \in \Omega$, uniformly for z belonging to any closed subset of Ω. Adding (2.10) and

(2.11) yields (2.2). ∎

Let $\phi^{-1}(z)$ denote the inverse map of $\phi(w)$. We introduce the inner product

$$(2.12) \qquad (f,g) := \frac{1}{2\pi} \int_{\Gamma} f(z)\overline{g(z)} |d\phi^{-1}(z)| = \frac{1}{2\pi} \int_{|w|=1} f(\phi(w))\overline{g(\phi(w))} |dw| .$$

The bar denotes complex conjugation.

Theorem 2.2.

$$(2.13) \qquad (p_n(z),w^{\ell}) = 0, \quad \ell > n, \quad w = \phi^{-1}(z) .$$

$$(2.14) \qquad (p_n(z),w^n) = 1 .$$

Proof.

$$(2.15) \quad \frac{1}{2\pi} \int_{|w|=1} \prod_{k=0}^{n-1} (\phi(w)-\phi(e^{2\pi i k/n}))\overline{w}^{-\ell} |dw| = \frac{1}{2\pi i} \int_{|w|=1} \prod_{k=0}^{n-1} (\phi(w)-\phi(e^{2\pi i k/n})) w^{-\ell-1} dw$$

ϕ being analytic exterior to $|w| = 1$, we can replace the integration path by a circle $|w| = R$ sufficiently large so that ϕ has an expansion

$$(2.16) \qquad \phi(w) = cw + a_0 + a_1 w^{-1} + a_2 w^{-2} + \ldots , \quad |w| > R .$$

Substituting (2.16) into (2.15) yields (2.13). (2.14) follows from

$$p_k(\phi(w)) = w^k + \sum_{j=-\infty}^{k-1} \alpha_{kj} w^j . \qquad \blacksquare$$

We will use a Petrov-Galerkin method to compute polynomial approximations. Let $G^{(n)}$ denote the Grammian

$$G^{(n)} = [G_{k\ell}], \quad G_{k\ell} := (p_{\ell}(z),w^k), \quad 0 < k,\ell < n, \quad w = \phi^{-1}(z) ,$$

and let $F^{(n)}$ be the Fourier operator

$$F^{(n)} f = [F_k], \quad F_k := (f(z),w^k), \quad 0 < k < n, \quad w = \phi^{-1}(z) .$$

Theorem 2.3.

Let $f(z)$ be analytic in Ω, and have uniformly bounded Fourier coefficients $F_k = (f(z),w^k)$, $z = \phi(w)$. Let the projection P_n be defined by

$$(2.17) \qquad P_n f := \sum_0^{n-1} a_k p_k ,$$

with $\underline{a} = (a_0, a_1, \ldots, a_{n-1})^T$, $\underline{a} := (G^{(n)})^{-1} F^{(n)} f$. Then, if $\frac{d^2\phi}{dw^2}$ is of bounded variation on $|w| = 1$,

(2.18) $$a_k + F_k, \quad k + \infty, \quad n > k .$$

Proof. By (2.13), (2.14) $G^{(n)}$ is an upper triangular matrix with diagonal elements $G_{kk} = 1$. The upper triangular elements have by theorem 2.1 the form $G_{\ell k} = (w^k + o(\frac{1}{k}), w^\ell) = \delta_{k\ell} + o(\frac{1}{k})$. Consider the vectors $\underline{a}^{(m)} = (a_m, a_{m+1}, a_{m+2}, \ldots, a_n)^T$, $\underline{b}^{(m)} = (F_m, F_{m+1}, \ldots, F_n)^T$, and let the matrix $E^{(m)} = [E_{kj}^{(m)}]$ be defined by

$$E_{kj}^{(m)} = \begin{cases} 1 & , \quad k = j , \quad 0 < k, j < m - n \\ G_{k+m, j+m} & , \quad k \neq j , \quad 0 < k, j < m - n \end{cases}$$

The magnitude of the $E_{kj}^{(m)}$ is given by

$$E^{(m)} = \begin{pmatrix} 0 & o(\frac{1}{m}) & o(\frac{1}{m+1}) & o(\frac{1}{m+z}) \cdots o(\frac{1}{n-1}) \\ & 0 & o(\frac{1}{m+1}) & o(\frac{1}{m+2}) \cdots o(\frac{1}{n-1}) \\ & & 0 & \cdots\cdots\cdots\cdots\cdots\vdots \\ 0 & & & \cdots\cdots 0 \end{pmatrix}$$

$|E^{(m)}|$ is bounded as $n + \infty$, and we can, for each $\varepsilon > 0$, select an m such that $|E^{(m)}|_\infty < \varepsilon$. Now

$$(I + E^{(m)}) \underline{a}^{(m)} = \underline{b}^{(m)} .$$

Let $|\underline{b}^{(m)}|_\infty < d$, and assume $\varepsilon < 1$. Then

$$\underline{a}^{(m)} - \underline{b}^{(m)} = \sum_{k=1}^{\infty} (E^{(m)})^k \underline{b}^{(m)} ,$$

and

$$|\underline{a}^{(m)} - \underline{b}^{(m)}|_\infty < (1 + |E^{(m)}|_\infty)^{-1} |E^{(m)}|_\infty |\underline{b}^{(m)}|_\infty < \frac{\varepsilon d}{1 - \varepsilon} . \qquad \blacksquare$$

Adding some regularity assumptions on the functions to be approximated, we can bound

(2.19) $$|P_n|_\infty = \sup_{|f|_\Gamma} |P_n f|_\Gamma ,$$

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where

$$\|f\|_\Gamma = \sup_{z \in \Gamma} |f(z)|$$

Theorem 2.4.

Let f be analytic on Ω, and assume its Fourier coefficients $(f(z), w^k)$, $w = \phi(z)$, $k = 0, 1, 2, \ldots$, form an absolutely convergent series. Let $\dfrac{d^2\phi}{dw^2}$ be of bounded variation on $|w| = 1$. Then P_n is bounded wrt the norm (2.19).

Proof. We divide the matrix $G^{(n)}$ into 2 parts. For an $m < n$, let

$$A = [A_{kj}], \quad A_{kj} = \begin{cases} G_{kj}, & k = j \text{ or } k < m \\ \\ 0, & \text{else .} \end{cases}$$

Let $E := G^{(n)} - A$. For an arbitrary $\epsilon > 0$, we can select an m so that $\|E\|_1 < \epsilon < 1$ for all n, c.f. the proof of theorem 2.3. Using $EA^{-1} = E$, we obtain

$$(G^{(n)})^{-1} = (A + E)^{-1} = A^{-1}(I + EA^{-1})^{-1} =$$

$$= A^{-1}(I + E)^{-1} = A^{-1}(I + \sum_1^\infty (-E)^k) = A^{-1} + A^{-1} \sum_1^\infty (-E)^k .$$

Hence,

$$(2.20) \qquad \underline{a} = A^{-1} F^{(n)} f + A^{-1} \sum_{k=1}^\infty (-E)^k F^n f .$$

$$\sup_{z \in \Gamma} \left| \sum_0^{n-1} a_k p_k(z) \right| < \sup_{|w|=1} \left| \sum_0^{n-1} a_k (w^k + o(\tfrac{1}{k})) \right|_\infty < \sum_0^{n-1} |a_k|(1 + o(\tfrac{1}{k})) < d_0 \|a\|_1$$

for some constant d_0 independent of n. From (2.20),

$$\|\underline{a}\|_1 < (1 + \epsilon)^{-1} \|A^{-1}\|_1 \cdot \|F^n f\|_1 . \qquad \blacksquare$$

$\|F^n f\|_1$ was assumed bounded for all n.

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3. COMPUTING WITH THE POLYNOMIALS

The proof of theorem 2.3 indicates a computational method for determining the coefficients of $P_n f$ in $O(n \log(n))$ operations if the Grammian is known.

1) Determine n Fourier coefficients $\underline{b} := F_n f$. This requires $O(n \log(n))$ operations.

2) Solve $G^{(n)} \underline{a} = \underline{b}$. Solutions of the full system requires $\frac{n^2}{2}$ operations, but we only need to solve $A\underline{a} = \underline{b}$, where A is the submatrix of $G^{(n)}$ introduced in the proof of theorem 2.3. A can be choosen independently of n. Solving $A\underline{a} = \underline{b}$ requires $O(n)$ operations.

We next turn to the computation of the polynomials. The restriction of the mapping function $\phi(w)$ to $|w| = 1$ is needed, and several numerical methods are available, see Fornberg [3], Gutknecht [5] or Reichel [6]. The method [6] yields also the capacity of Γ, but not knowing the capacity only necessitates explicit normalization $(P_k(z), (\phi^{-1}(z))^k) = 1$, $k = 0, 1, \ldots, n - 1$. We finally note that when $P_n f$ has been computed and is to be evaluated at many points it might be advantageous to use a representation which is faster to evaluate than (2.17), like a Newton polynomial representation.

4. NUMERICAL EXAMPLES

We consider two contours Γ, one which is analytic, and one for which $\dfrac{d^2\phi}{dw^2}$ has a jump discontinuity on $|w| = 1$. All computations have been carried out on a UNIVAC 1100 in single precision, i.e. with 8 significant digits. The images of the roots of unity, we determined with approximately 6 significant digits. Let Γ be the ellipse
$\{x + iy : \left(\frac{x}{2}\right)^2 + y^2 = 1\}$.

Ex. E1.

j	$\max\limits_{\|w\|=1} \|P_{j+1}(\phi(w)) - w^{j+1}\|$
5	$4.1 \cdot 10^{-3}$
10	$2.4 \cdot 10^{-5}$

Due to rounding errors, we cannot obtain a deviation much smaller than for $j = 10$. ∎

Ex. E2. Let $f(z) := (z + 3)^{-1}$.

| j	$\sup\limits_{z \in \Gamma} \|(P_{j+1}f)(z) - f(z)\|$
10	$1.3 \cdot 10^{-3}$
20	$3.3 \cdot 10^{-6}$

If the error would decrease maximally, see Gaier [4], ch. 1, it would decrease by a factor $2.56 \cdot 10^{-3}$, when j is increased from 10 to 20. This is also the case. When j is increased further, rounding errors dominate. ∎

Ex. E3. Let $f(z) := \sqrt{z + 2}$, where we choose a branch which has a discontinuity on the negative real axis.

j	$\sup\limits_{z \in \Gamma} \|(P_{j+1}f)(z) - f(z)\|$
10	0.176
20	0.121
40	0.079

The error decreases by a factor close to $\frac{1}{\sqrt{2}}$, the expected rate of convergence. ▪

In the following examples Γ is a sports ground shaped region obtained by placing a unit square between 2 unit disk halves.
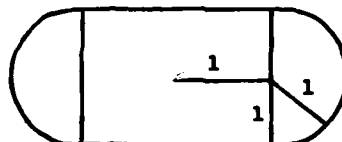
Ex. S1

j	$\max\limits_{|w|=1} |P_{j+1}(\phi(w)) - w^{j+1}|$
5	$8.86 \cdot 10^{-2}$
10	$2.82 \cdot 10^{-2}$
20	$1.24 \cdot 10^{-2}$
40	$0.61 \cdot 10^{-2}$

The error seems to decrease like $o(1/n)$, $n \to \infty$. ▪

Ex. S2 $f(z) := (z + 3)^{-1}$

j	$\max\limits_{z \in \Gamma} |(P_{j+1}f)(z) - f(z)|$
10	$2.4 \cdot 10^{-3}$
20	$1.0 \cdot 10^{-5}$

▪

Ex. S3 $f(z) := \sqrt{z + 2}$, the same branch as in ex. E3

j	$\max\limits_{z \in \Gamma} |(P_{j+1}f)(z) - f(z)|$
10	0.198
20	0.137
40	0.090

▪

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